## EQUATION OF CREEP OF MATERIALS LIABLE

TO STRAIN-HARDEN DURING CREEP

## A. F。Nikitenko

An expression in the form

$$
\begin{equation*}
p^{\boldsymbol{\alpha}} d p=K_{0} e^{\beta_{2} \sigma} d t \tag{1}
\end{equation*}
$$

is widely used to describe transient creep phenomena in the framework of the theor $y$ of strain hardening.
Here p is creep strain, $\sigma$ is stress, t is time, and $\alpha, \mathrm{K}_{0}$, and $\beta_{0}$ are material constants at a given temperature and in a given stress range. However, it was observed in [1] that $\alpha$ and $\beta_{0}$ do not remain constant when the applied stress varies within wide limits. In this article this problem is analyzed in more detail on the basis of experimental data cited in [2-5].

The material characteristics are usually determined in the following way [2]: integrating (1) for $\sigma=$ const and taking $1 /(1+\alpha)=m$, we obtain the equation

$$
\begin{equation*}
p=\frac{K_{0}}{m} \mathrm{e}^{m \beta_{0} \mathrm{~s}} t^{m} \tag{2}
\end{equation*}
$$

which in a system of coordinates $\log p-\log t$ is represented by a set of straight lines with parameter $\sigma$. In $[2,4,5]$ but not in [3] these straight lines were almost parallel, which makes it possible to conclude that $\alpha=$ const in the entire stress range under consideration.

Considering the values of creep strain corresponding to different stress levels at a certain time $t$, from (2) we obtain

$$
\begin{equation*}
\beta_{0}=\frac{\lg p_{j}-\lg p_{i}}{0.4343 m\left(\sigma_{j}-\sigma_{i}\right)} \tag{3}
\end{equation*}
$$

from which the average value of $\beta_{0}$ is found.
Analysis of experimental data in [2-5] showed that $\beta_{0}$ monotonically increases with $\sigma$. Values of $\beta_{0}$ calculated from (3) for stresses $\sigma=1 / 2\left(\sigma_{j}+\sigma_{i}\right) 12$ are plotted in Fig. 1, curves 1 and 2, relating, respectively, to data from [4] and [5]. We see that the statement $\beta_{0}=$ const is a very rough approximation of reality. It therefore seems advisable to regard $\beta_{0}$ as a function (e.g., a power function) of stress and replace (1) by an equation in the form

$$
\begin{equation*}
p^{\alpha} d p=K \mathrm{e}^{\beta \sigma^{n}} d t \quad(n>1) \tag{4}
\end{equation*}
$$

and Eq. (2) by

$$
\begin{equation*}
p=\frac{K}{m} \mathrm{e}^{m \beta \sigma^{n}} t^{m} \tag{5}
\end{equation*}
$$

Taking the logarithm of (5), we obtain

$$
\begin{equation*}
\lg p=0.4343 m 3 \sigma^{n}+\lg \left(\frac{K}{m} t^{m}\right) \tag{6}
\end{equation*}
$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 3, pp. 183-184, May-June, 1969. Original article submitted September 24, 1966.

[^0]

Fig． 1


Fig。2

If the process of creep is described by（4），then experimental points plotted in coordinates $\log p-\sigma^{n}$ should lie on one of a set of straight lines with a parameter $t$ ．The index $n$ is determined from the condi－ tion stipulating that three experimental points representing the state $\sigma_{1}, \mathrm{p}_{1} ; \sigma_{2}, \mathrm{p}_{2} ; \sigma_{3}, \mathrm{p}_{3}$ ；at a certain time t （where $\sigma_{1}<\sigma_{2}<\sigma_{3}$ ）should lie on one straight line

$$
\begin{equation*}
\left(\sigma_{2}^{n}-\sigma_{1}^{n}\right) \lg p_{3}+\left(\sigma_{3}^{n}-\sigma_{2}^{n}\right) \lg p_{1}-\left(\sigma_{3}^{n}-\sigma_{1}^{n}\right) \lg p_{2}=0 \tag{7}
\end{equation*}
$$

When data from［2－5］were processed，it was found that experimental points plotted in coordinates $\log p-\sigma^{n}$ at $t=$ const do，in fact，lie on a set of almost parallel straight lines．For instance，analysis of data from［5］showed that the variation in $\beta$ determined in different intervals（ $\sigma_{\mathrm{i}}, \sigma_{j}$ ）from（6）with the aid of a formula

$$
\begin{equation*}
\beta=\frac{\lg p_{j}-\lg p_{i}}{0.4343 m\left(\sigma_{j}^{n}-\sigma_{i}^{n}\right)} \tag{8}
\end{equation*}
$$

does not exceed $\pm 5 \%$ ．
Solving Eq．（7）gave the following values of $n$ ：

$$
\begin{array}{ll}
\text { for duralumin D16T at } 150^{\circ} \mathrm{C}[2] & \mathrm{n}=2.817, \\
\text { for high-quality structural } 30 \mathrm{CrMo} \text { at } 500^{\circ} \mathrm{C}[3] & \mathrm{n}=2.188 \\
\text { for Nimonic-75 at } 650^{\circ} \mathrm{C}[4] & \mathrm{n}=1.495 \\
\text { and for an aluminum alloy at } 200^{\circ} \mathrm{C}[5] & \mathrm{n}=2.620 .
\end{array}
$$

The constant $K$ is determined by the previous method［2］．
Creep curves calculated from（4）are in very good agreement with experimental data。Figures cited above show that $n$ oscillates about values whose mean is approximately 200 ．Since the introduction of an additional material constant in（4）is，generally speaking，undesirable，it is advisable（in view of the result obtained）to take $n=2$ in（4），thereby leaving unchanged the method of determining material characteristics ［2］．

The results of processing data from［2］are reproduced in Fig。2，where the dashed lines represent curves calculated from（4）with $n=2.620$ ，the solid lines represent curves calculated from（4）with $n=2$ ， the dot－dash lines are curves calculated from（1），and the numbers indicate stress levels（in $\mathrm{kg} / \mathrm{mm}^{2}$ ）at which creep tests were carried out．

Thus，without complicating the method of determining material constants，curves calculated from（4） with $n=2$ give a substantially better approximation than those calculated from（1）．

Thanks are due to $M_{0}$ V．Mitrofanov and G．Ya．Sofienko for their assistance in the processing of experimental data and to $\mathrm{O} . \mathrm{V}_{0}$ ．Sosnin who sponsored this work．

## LITERATURE CITED

1．O．V．Sosnin and N．G．Torshenov，＂Compression and buckling of columns in creep under mono－ tonically increasing loads，＂PMTF［Journal of Applied Mechanics and Technical Physics］，no．5， 1967.
2．V．S．Namestnikov and A．A．Khvostunkov，＂Creep of duralumin under constant and variable loads，＂ PMTF，［Journal of Applied Mechanics and Technical Physics］，no．4， 1960.
3. V. I. Danilovskaya, G. M. Ivanova, and Yu. N. Rabotnov, "Creep and relaxation of chromium-molybdenum steel," Izv. AN SSSR, OTN, no. 5. 1955.
4. A. E. Johnson, "Creep under complex stress systems at elevated temperatures," Proc. Inst. Mech. Engr., vol. 164, no. 4, 1951.
5. A. E. Johnson, "The creep of a nominally isotropic aluminum alloy under combined stress systems at elevated temperatures, ${ }^{\text {n }}$ Metallurgia, vol. 40, pp. 125-139, July, 1949.


[^0]:    © 1972 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for $\$ 15.00$.

